

2.1 Exercise Set

1. Fill in the blanks to make the statement true.

- a) In a rational number or rational expression, the term on the top is called the numerator, and the term on the bottom is called the denominator.
- b) The statement $\frac{3}{x}$ is called a rational expression.
- c) The denominator in a rational expression cannot be zero.
- d) Division by 0 is not permissible; undefined.

2. Simplify the following rational numbers.

- a) $\frac{21}{63} \div \frac{21}{21}$ $\frac{1}{3}$
- b) $-\frac{36}{84} \div \frac{12}{12}$ $-\frac{3}{7}$
- c) $\frac{65}{78} \div \frac{13}{13}$ $\frac{5}{6}$
- d) $-\frac{132}{2310} \div \frac{11}{11} = \frac{12}{210} \div \frac{6}{6}$ $-\frac{2}{35}$

3. Determine the restriction(s) on the following rational expressions.

- a) $\frac{3}{x}$ $x \neq 0$
- b) $\frac{x+2}{x-1}$ $x \neq 1$
- c) $\frac{x-1}{x^2+1}$ no restr.
- d) $\frac{x(x+2)}{(x+2)(x-3)}$ $x \neq -2, 3$
- e) $\frac{x^2-x-6}{x^2-4x}$ $x \neq 0, 4$
- f) $\frac{x^2-x-6}{x^2-4x+4}$ $x \neq 2$
- g) $\frac{x^2+4}{x^2-3x+2}$ $x \neq 1, 2$
- h) $\frac{x^2+y^2}{x^2-y^2}$ $x \neq \pm y$
- i) $\frac{x-y}{x^2-2xy+y^2}$ $x \neq y$
- j) $\frac{x^2-4}{x^4-1}$ $x \neq \pm 1$

4. Simplify the following rational expressions. Assume no denominators are zero.

a) $\frac{x-1}{x-1}$

$$\frac{1}{1}$$

b) $\frac{x-1}{1-x} \cdot \frac{x-1}{-(x-1)}$

$$\frac{-1}{1}$$

c) $\frac{x+1}{1+x}$

$$\frac{1}{1}$$

d) $\frac{x+2}{-2-x} \cdot \frac{x+2}{-(x+2)}$

$$\frac{-1}{1}$$

e) $\frac{(x-1)(x+1)}{(1-x)(-x-1)}$

$$\frac{1}{1}$$

f) $\frac{3-x}{x+3}$

$$\frac{3-x}{x+3}$$

g) $\frac{(x-2)(x-1)(x+2)}{(2-x)(1-x)(2-x)}$

$$\frac{x+2}{2-x}$$

h) $\frac{x^2-4}{4-x^2} \cdot \frac{(x-2)(x-2)}{(2-x)(2+x)}$

$$\frac{-1}{1}$$

i) $\frac{(x-1)(x-1)}{-x^2+2x-1} \cdot -1$

$$\frac{-1}{1}$$

j) $\frac{a-b+c}{b-c-a} \cdot -1$

$$\frac{-1}{1}$$

5. Simplify the following rational expressions. Assume no denominators are zero.

a) $\frac{8x}{14y} \cdot \frac{2}{7}$

$$\frac{4x}{7y}$$

b) $\frac{5}{24x} \cdot \frac{18xy}{8}$

$$\frac{5y}{8}$$

c) $\frac{3}{5} \cdot \frac{6(x+2)}{10(x+2)}$

$$\frac{3}{5}$$

d) $\frac{(x+1)(x-1)}{(x-1)(x-1)}$

$$\frac{x+1}{x-1}$$

e) $\frac{x^2-xy}{x^2} \cdot \frac{x(x-y)}{x}$

$$\frac{x-y}{x}$$

f) $\frac{2x^2-8x}{4-x} \cdot \frac{2x(x-4)}{4-x}$

$$\frac{-2x}{1}$$

5. g) $\frac{x^2 \cancel{x}}{x^3 + x^2 y} \cdot \frac{\cancel{x}}{x+4}$ h) $\frac{4x^2 + 16x}{x^2 - 16} \cdot \frac{4\cancel{x}(x+4)}{(x+4)(x-4)} \cdot \frac{4\cancel{x}}{x-4}$
- i) $\frac{x(x+2)}{x^2 + 3x + 2} \cdot \frac{x}{x+1}$ j) $\frac{(x+6)(x+3)}{x^2 + 9x + 18} \cdot \frac{x+3}{x}$
- k) $\frac{(2x-1)(x+3)}{x^2 - 9} \cdot \frac{2x-1}{x-3}$ l) $\frac{(3x+2)(x+4)}{x^2 - 16} \cdot \frac{3x+2}{x+4}$
- m) $\frac{(2x+7)(x+5)}{3x^2 + 19x + 20} \cdot \frac{2x+7}{3x+4}$ n) $\frac{(5x-2)(x+6)}{4x^2 - 27x + 18} \cdot \frac{5x-2}{4x-3}$
- o) $\frac{(7x-2)(x+9)}{7x^2 + 19x - 6} \cdot \frac{x+9}{x+3}$ p) $\frac{(8x-3)(x-6)}{8x^2 + 29x - 12} \cdot \frac{x-6}{x+4}$
- q) $\frac{3(x-7)}{28-4x} \cdot \frac{-3}{4}$ r) $\frac{(x+9)(x-2)}{12-4x-x^2} \cdot \frac{-x+9}{x+6}$
- s) $\frac{(x-4)(x+34)}{2x^2 - xy - y^2} \cdot \frac{x+34}{2x+y}$ t) $\frac{(x-14)(x-24)}{x^2 - 3xy + 2y^2} \cdot \frac{x-4}{x+24}$
- u) $\frac{(2x+4)(x-4)}{(2x-3)(x+2)} \cdot \frac{x+2}{5x+4}$ v) $\frac{(2x+5)(x-3)}{(2x^2 - x - 15)} \cdot \frac{-2x+5}{x-7}$
- w) $\frac{(6-x-2x^2)}{12+7x-10x^2} \cdot \frac{x+2}{5x+4}$ x) $\frac{(x-7)(x-3)}{(x-1)(x+4)} \cdot \frac{x-1}{x-4}$
- y) $\frac{(2x-3)(5x+4)}{x(4-3) \cdot 2(4-3)} \cdot \frac{y-3}{y+5}$ z) $\frac{(x-1)(y+4)}{xy+4x+y-4} \cdot \frac{x-1}{x-4}$
- aa) $\frac{(x+2)(y-3)}{(x+2)(y+5)} \cdot \frac{x-1}{x-4}$

6. Explain the error.

a) $\frac{2(x-1)-x}{x-1} = 2-x$

can't cancel!

$$= \frac{2x-2-x}{x-1}$$

$$= \frac{x-2}{x-1}$$

b) $\frac{3x^2-12}{3x-3} = x-4$

can't divide

$$= \frac{3(x^2-4)}{3(x-1)}$$

$$= \frac{3(x+2)(x-2)}{3(x-1)}$$

7. Reduce the rational expression.

a) $\frac{(a+b)^2 - (x+y)^2}{a+b+x+y}$

$$\frac{(a+b+x+y)(a+b-x-y)}{a+b+x+y}$$

$$= a+b-x-y$$

b) $\frac{x-1}{x^3-x^2-x+1}$

$$\frac{x^2(x-1)-1(x-1)}{(x^2-1)(x-1)}$$

$$\frac{(x+1)(x-1)(x-1)}{(x+1)(x-1)(x-1)}$$

$$= \frac{1}{x-1}$$

c) $\frac{x^{2a} + 3x^a - 18}{x^{2a} - 36}$

$$\frac{(x^a+6)(x^a-3)}{(x^a+6)(x^a-6)} = \frac{x^a-3}{x^a-6}$$

d) $\frac{2x^{2a+1} + 3x^{a+1} + x}{4x^{2a+1} - x}$

$$= \frac{x(2x^{2a} + 3x^a + 1)}{x(4x^{2a} - 1)}$$

$$= \frac{x^a + 1}{2x^a - 1}$$

e) $\frac{x^{2b+1} - x}{x^{b+1} - x}$

$$\frac{x^{2b} \cdot x - x}{x^b \cdot x - x}$$

$$\frac{x(x^{2b} - 1)}{x(x^b - 1)} = \frac{(x^b+1)(x^b-1)}{x^b-1}$$

$$= x^b + 1$$

f) $\frac{3x^{2a+2} + 7x^{a+2} - 6x^2}{6x - 7x^{a+1} - 3x^{2a+1}}$

$$\frac{x^2 \cdot 3x^{2a} + x^2 \cdot 7x^a - 6x^2}{6x - 7x \cdot x^a - 3x \cdot x^{2a}}$$

$$\frac{x^2(3x^{2a} + 7x^a - 6)}{x(6 - 7x^a - 3x^{2a})} = -x$$

8. The volume of a box is
- $(6x^2 + 23x + 21)(x + 5)$
- . If the height of the box is
- $2x + 3$
- , what is the area of the bottom of the box?
- $\rightarrow l \cdot w$

$$\frac{(6x^2 + 23x + 21)(x + 5)}{(2x + 3)}$$

area

$$A = 3x^2 + 22x + 35 \text{ u}^2$$

9. The volume of a box is
- $(2x^{2a} + x^a - 1)(x^a - 1)$
- . If the height of the box is
- $x^a + 1$
- , what is the area of the bottom of the box?
- $\rightarrow l \cdot w$

$$\frac{(2x^{2a} + x^a - 1)(x^a - 1)}{(x^a + 1)}$$

h

$$A = (2x^a - 1)(x^a - 1)$$

$$A = 2x^{2a} - 3x^a + 1 \text{ u}^2$$

2.2 Exercise Set

1. Fill in the blanks to make the statement true.

a) If $\frac{a}{b}$ and $\frac{c}{d}$ are polynomials with $b, d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

b) If $\frac{a}{b}$ and $\frac{c}{d}$ are polynomials with $b, c, d \neq 0$ then $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

c) The reciprocal of $\frac{x-y}{x+y}$ is $\frac{x+y}{x-y}$

d) To multiply two rational expressions, the numerator and denominator are written as a single expression then simplify each expression.e) To divide two rational expressions, the problem is re-written as the product of the dividend and the reciprocal of the divisor.

2. Multiply. Simplify answers if possible. Assume the denominators do not equal zero.

a) $\frac{3 \cancel{15x^4} \cancel{8}}{18 \cancel{2x}} = \frac{3x}{2}$

b) $\frac{4 \cancel{8(x-2)}}{y \cancel{8(x-2)^2}} = \frac{4}{x-2}$

c) $\frac{\cancel{3} \cancel{2x-6}}{2(\cancel{x-3})} \cdot \frac{\cancel{x-3}}{\cancel{6}} = \frac{1}{4}$

d) $\frac{\cancel{8(x-3)} \cdot \cancel{4(x+3)}^2}{\cancel{3x+9} \cdot \cancel{6x-18}} = \frac{16}{9}$

e) $\frac{\cancel{x-2}^{-1}}{\cancel{8}} \cdot \frac{\cancel{6}^3}{\cancel{2-x}} = -\frac{3}{4}$

f) $\frac{\cancel{3(8-x)}^{-1}}{\cancel{4}} \cdot \frac{\cancel{12}^3}{\cancel{2(x-8)}} = -\frac{9}{2}$

g) $\frac{\cancel{x-2y}^{-1}}{\cancel{6}} \cdot \frac{\cancel{3}}{\cancel{2y-x}} = -\frac{1}{2}$

h) $\frac{\cancel{15}^5 \cancel{2y-3x}^{-1}}{\cancel{3x-2y}} \cdot \frac{\cancel{12}}{\cancel{4}} = -\frac{5}{4}$

$$2. \quad i) \quad \frac{\cancel{(x+2)}\cancel{(x-2)}}{2} \cdot \frac{3\cancel{(x^2-4)}}{28\cancel{(x-2)}} \cdot \frac{14x}{11\cancel{(x+2)}} = \frac{3x}{22}$$

$$j) \quad \frac{\cancel{(x-2)}\cancel{(x+1)}}{x+3} \cdot \frac{3\cancel{(x+3)}}{2x+2} = \frac{3(x-2)}{2}$$

$$k) \quad \frac{\cancel{(x-5)}\cancel{(x+1)}}{x^2-7x+10} \cdot \frac{x-4}{x+1} = \frac{x-4}{x-2}$$

$$l) \quad \frac{\cancel{(y+2)}\cancel{(y+1)}}{y^2-4y+3} \cdot \frac{y-1}{y+1} = \frac{y+2}{y-3}$$

$$m) \quad \frac{\cancel{(2z+1)}\cancel{(z-1)}}{2z^2+5z+3} \cdot \frac{2z^2+z-3}{4z^2-1} = \frac{(z-1)^2}{(z+1)(2z-1)}$$

$$n) \quad \frac{\cancel{x}\cancel{(4x-1)}}{6x^2+10x} \cdot \frac{3x^2+11x+10}{8x^2+2x-1} = \frac{x+2}{2(2x+1)}$$

$$o) \quad \frac{\cancel{(y+5)}\cancel{(y-1)}}{4y^2-9} \cdot \frac{2y^2-5y-12}{y^2+y-20} = \frac{y-1}{2y-3}$$

$$p) \quad \frac{\cancel{(z-6)}\cancel{(z-3)}}{4z^2-9} \cdot \frac{2z^2-5z-12}{z^2-10z+24} = \frac{z-3}{2z-3}$$

$$q) \quad \frac{\cancel{(x+4y)}\cancel{(x-2y)}}{x^2+4xy+3y^2} \cdot \frac{x^2+2xy-3y^2}{x^2-3xy+2y^2} = \frac{x+4y}{x+y}$$

$$r) \quad \frac{\cancel{(x-3y)}\cancel{(x+2y)}}{x^2-4xy+3y^2} \cdot \frac{y^2-x^2}{x^2+3xy+2y^2} = -1$$

$$s) \quad \frac{\cancel{(z+3)}\cancel{(z-3)}}{z^2+7z+12} \cdot \frac{z^2-2z-8}{z^2-z-6} = \frac{z-4}{z+4}$$

$$t) \quad \frac{x+3}{x-1} \cdot \frac{x^2+4x-12}{x^2+x-6} = \frac{x+6}{x-1}$$

$$u) \quad \frac{\cancel{(2y+1)}\cancel{(y+5)}}{2y^2+7y+3} \cdot \frac{y^2-9}{y^2-3y} = \frac{y+5}{y}$$

$$v) \quad \frac{\cancel{(x+6y)}\cancel{(x-5y)}}{x^2-xy-20y^2} \cdot \frac{x^2+xy-12y^2}{x^2-2xy-3y^2} = \frac{x+6y}{x+y}$$

3. Divide. Simplify answers if possible. Assume the denominators do not equal zero.

$$a) \frac{5x^2}{21} \div \frac{15x^6}{14} = \frac{\cancel{5}x^2}{\cancel{21}^3} \times \frac{\cancel{14}^2}{\cancel{15}^3x^6} = \frac{2}{9x^4}$$

$$b) \frac{12}{2x-3y} \div \frac{15}{3y-2x} = \frac{\cancel{12}^4}{\cancel{2x-3y}^4} \cdot \frac{\cancel{3y-2x}^{-1}}{\cancel{15}^5} = -\frac{4}{5}$$

$$c) \frac{x^2-x}{x} \div (3x-3) = \frac{x(x-1)}{x} \cdot \frac{1}{3(x-1)} = \frac{1}{3}$$

$$d) \frac{5y-10}{y+2} \div \frac{3y-6}{2} = \frac{5(y-2)}{y+2} \cdot \frac{2}{3(y-2)} = \frac{10}{3(y+2)}$$

$$e) \frac{2z-2}{2z-4} \div (z^2-z) = \frac{2(z-1)}{2(z-2)} \cdot \frac{1}{z(z-1)} = \frac{1}{z(z-2)}$$

$$f) \frac{y^2+3y}{9} \div \frac{y+3}{3y} = \frac{y(y+3)}{3 \cdot 3} \cdot \frac{3y}{y+3} = \frac{y^2}{3}$$

$$g) \frac{6z}{z-2} \div \frac{3z^2}{(z-2)^2} = \frac{\cancel{6}^2}{\cancel{z-2}} \cdot \frac{(z-2)^2}{3z^2} = \frac{2(z-2)}{z}$$

$$h) \frac{3x^2-20x-7}{x^2-2x-35} \div (3x^2-14x-5) = \frac{(3x+1)(x-7)}{(x-7)(x+5)} \cdot \frac{1}{(3x+1)(x-5)(x+5)(x-5)} = \frac{1}{(x+5)(x-5)^2}$$

$$i) \frac{4a^2-ab-5b^2}{ax+by+ay+bx} \div (4a-8b) = \frac{(4a-5b)(a+b)}{a(x+b)+b(x+a)} \cdot \frac{1}{4(a-2b)} = \frac{4a-5b}{4a(x+b)(a-2b)}$$

$$j) \frac{y^2-10y+9}{y^2-1} \div \frac{y^2-5y-36}{1-y^2} = \frac{(y-9)(y-1)}{(y+1)(y-1)} \cdot \frac{(1-y)(1+y)}{(y-9)(y+4)(y+4)} = \frac{1-y}{y+4}$$

$$k) \frac{z^2-6z+9}{12-4z} \div \frac{z^3-3z^2}{z^6-9z^4} = \frac{(z-3)(z-3)}{4(3-z)} \cdot \frac{z^4(z+3)(z-3)}{z^2(z-3)} = \frac{-z(z-3)(z+3)}{4}$$

$$l) \frac{x(x^2-4)}{x-x^4} \div \frac{4x-x^3}{x^4-x} = \frac{x(x+2)(x-2)}{x(1-x^3)} \cdot \frac{x(1-x^3)}{x(2+x)(2-x)} = 1$$

$$3. - (m) \frac{4y-4x}{8y^3} \div \frac{x^2-y^2}{2x+2y}$$

$$\frac{\cancel{4}(\cancel{y-x})}{\cancel{8}y^3} \cdot \frac{\cancel{2}(\cancel{y+x})}{(x+y)(x-y)} = -\frac{1}{y^3}$$

$$n) \frac{x^2-y^2}{3x^2+3xy} \div \frac{3x^2-2xy-y^2}{3x^2+6x}$$

$$\frac{(\cancel{x+y})(\cancel{x-y})}{3x(\cancel{x+y})} \cdot \frac{\cancel{3}x(\cancel{x+2})}{(3x+4)(\cancel{x-y})}$$

$$= \frac{x+2}{3x+4}$$

$$o) \frac{3x+4y}{x^2+4xy+4y^2} \div \frac{2}{x+2y}$$

$$\frac{3x+4y}{(x+2y)^2} \cdot \frac{\cancel{x+2y}}{2}$$

$$= \frac{3x+4y}{2(x+2y)}$$

$$p) \frac{x^2-4}{2y} \div \frac{2-x}{6xy}$$

$$\frac{(x+2)(\cancel{x-2})}{2y} \cdot \frac{\cancel{6}xy}{\cancel{2-x}}$$

$$= -3x(x+2)$$

$$x(x+1)-4(y+1) \quad 5(x+y+1)$$

$$q) \frac{3x^2-x}{6x^2+15x} \div \frac{6x^2+x-1}{2x^2-5x-25}$$

$$\frac{x(3x-1)}{3x(2x+5)} \cdot \frac{(2x+5)(x-5)}{(3x-1)(2x+1)}$$

$$= \frac{x-5}{3(2x+1)}$$

$$r) \frac{x^2+x-y^2-y}{3x^2-3y^2} \div \frac{5x+5y+5}{7x^2y+7xy^2}$$

$$\frac{3(x^2-y^2)}{3x(x-y)} \cdot \frac{7xy(x+y)}{7xy(x+y)} = \frac{7xy}{15}$$

$$t) \frac{x-8}{x-4} \div \left(\frac{x^2-12x+32}{8x} \cdot \frac{x^2-8x}{x^2-8x+16} \right)$$

$$\frac{x-8}{x-4} \cdot \frac{8x(x-4)^2}{(x-8)(x-4)x(x-4)}$$

$$= \frac{8}{x-8}$$

$$s) \frac{x+5}{x+10} \div \left(\frac{10x}{x^2+10x} \cdot \frac{x^2+10x+25}{x^2+15x+50} \right)$$

$$\frac{\cancel{x+5}}{\cancel{x+5}} \cdot \frac{\cancel{x}(\cancel{x+10})(\cancel{x+10})(x+5)}{10\cancel{x}(\cancel{x+5})(\cancel{x+5})}$$

$$= \frac{x+10}{10}$$

$$u) \frac{3x-3y-x^2+y^2}{4x^2-4xy+y^2} = \frac{x(3-y)-y(3-y)}{(2x-y)^2}$$

$$= \frac{(x-y)(3-x-y)}{(2x-y)^2}$$

$$v) \frac{4x^2-y^2+10x-5y}{2x^2+xy+5x} \rightarrow x(2x+y+5)$$

$$\frac{4x^2+10x-y^2-5y}{2x(x+5)-y(y-5)} = \frac{(2x+y+5)(2x-y)}{x(2x+y+5)}$$

$$= \frac{2x-y}{x}$$

$$w) \frac{x^{2a}+x^a-6}{x^{2a}+6x^a+9} \div \frac{x^{2a}-4}{x^{2a}+2x^a-3}$$

$$\frac{(\cancel{x^a+3})(\cancel{x^a-2})}{(\cancel{x^a+3})(\cancel{x^a+3})} \cdot \frac{(\cancel{x^a+3})(\cancel{x^a-1})}{(x^a+2)(x^a-2)}$$

$$= \frac{x^a-1}{x^a+2}$$

$$x) \frac{x^{2b}+2x^b-8}{x^{2b}+3x^b-4} \div \frac{x^{2b}-x^b-2}{x^{2b}-1}$$

$$\frac{(\cancel{x^b+4})(\cancel{x^b-2})}{(\cancel{x^b+4})(\cancel{x^b-1})} \cdot \frac{(\cancel{x^b+1})(\cancel{x^b-1})}{(\cancel{x^b-2})(\cancel{x^b+1})}$$

$$= 1$$

2.3 Exercise Set

1. True or false.

a) $\frac{x}{x+2} - \frac{x+1}{x+2} = \frac{-1}{x+2}$

b) $\frac{(x+2)(x-3)}{(x-3)(x+2)} = 0$

T/F

c) $\frac{x+1}{x-3} + \frac{x-3}{x+1} = 2$

d) $\frac{x+3}{x-2} + \frac{x-4}{x+3} = \frac{2x-1}{2x+1}$

T/F

e) $\frac{x+3}{2x-5} - \frac{x+5}{2x-5} = \frac{8}{2x-5} - 2$

f) $\frac{3}{x} + \frac{5}{x} = \frac{3+5}{x+x}$

T/F

g) $\frac{1}{x+2} = \frac{1}{x} + \frac{1}{2}$

h) $\frac{3}{x+1} + \frac{3}{x-1} = \frac{5}{2x}$

T/F

2. Find the LCD for each rational expression.

a) $\frac{3}{8}, \frac{5}{72}$

72

b) $\frac{3}{x^2}, \frac{5}{x^5}$

x^5

c) $\frac{2}{3x^2}, \frac{5}{6x}$

$6x^2$

d) $\frac{-2}{3x^3y^4}, \frac{5}{9x^5y^6}$

$9x^5y^6$

e) $\frac{5}{6x}, \frac{13}{4x-8}$
 $4(x-2)$

f) $\frac{5}{8x}, \frac{-3}{12x-12}$
 $12x(x-2)$

$24x(x-1)$

g) $\frac{10}{27x^3}, \frac{5}{9x-45}$
 $9(x-5)$

$27x^3(x-5)$

h) $\frac{2}{4x-8}, \frac{5}{5x-10}$
 $4(x-2)(x-2)$

$20(x-2)$

i) $\frac{3}{4x^2+12x}, \frac{7}{3x^2+9x}$
 $4x(x+3)(x+3)$

j) $\frac{2}{x-y}, \frac{3}{y-x}$
 $12x(x+3)$

$x-y$

k) $\frac{5}{x-3}, \frac{-2}{3-x}$

$x-3$

l) $\frac{1}{4x-8}, \frac{-3}{x^2+2x-8}$
 $4(x-2)(x+4)(x-2)$

$4(x-2)(x+4)$

3. Perform the indicated operation. Simplify when possible. Assume the denominators are not zero.

$$\begin{aligned} \text{a) } \frac{3x}{x+1} + \frac{3}{x+1} &= \frac{3x+3}{x+1} \\ &= \frac{3(x+1)}{x+1} = 3 \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{6y}{y-2} - \frac{12}{y-2} &= \frac{6y-12}{y-2} = \frac{6(y-2)}{y-2} \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{14x}{2x+3y} + \frac{21y}{2x+3y} &= \frac{14x+21y}{2x+3y} \\ &= \frac{7(2x+3y)}{(2x+3y)} = 7 \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{z-2}{5z+3} - \frac{6z-5}{5z+3} &= \frac{z-2-6z+5}{5z+3} \\ &= \frac{-5z+3}{5z+3} \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{9x-1}{3x-5} - \frac{3x+9}{3x-5} &= \frac{9x-1-3x-9}{3x-5} \\ &= \frac{6x-10}{3x-5} = \frac{2(3x-5)}{3x-5} = 2 \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{-15y}{1-5y} - \frac{-3}{(5y-1)} &= \frac{-15y+3}{1-5y} = \frac{3(-5y+1)}{1-5y} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{g) } \frac{2z}{3z-1} + \frac{-z}{(1-3z)} &= \frac{2z}{3z-1} - \frac{z}{3z-1} \\ &= \frac{z}{3z-1} \end{aligned}$$

$$\begin{aligned} \text{h) } \frac{-8x}{(6x-5)} + \frac{10-4x}{5-6x} &= \frac{10-12x}{5-6x} \\ &= \frac{2(5-6x)}{5-6x} = 2 \end{aligned}$$

$$\begin{aligned} \text{i) } \frac{3y-2}{y^2-25} - \frac{4y-7}{y^2-25} &= \frac{-y+5}{(y+5)(y-5)} \\ &= \frac{-1}{y+5} \end{aligned}$$

$$\begin{aligned} \text{j) } \frac{2z-5}{z^2-9} - \frac{3z-8}{z^2-9} &= \frac{2z-5-3z+8}{z^2-9} \\ &= \frac{-z+3}{z^2-9} = \frac{-z+3}{(z+3)(z-3)} = \frac{-1}{z+3} \end{aligned}$$

$$\begin{aligned} \text{k) } \frac{x}{x-y} - \frac{y}{x-y} &= \frac{x-y}{x-y} = 1 \end{aligned}$$

$$\begin{aligned} \text{l) } \frac{x^2-8x}{x-5} - \frac{-15}{(5-x)} &= \frac{x^2-8x+15}{x-5} = \frac{(x-5)(x-3)}{x-5} = x-3 \end{aligned}$$

$$\begin{aligned} \text{m) } \frac{-x^2}{(x^2-y^2)} - \frac{y^2}{y^2-x^2} + \frac{2xy}{y^2-x^2} &= \frac{-x^2-y^2+2xy}{y^2-x^2} \\ &= \frac{-(x^2+y^2-2xy)}{(y+x)(y-x)} = \frac{-(x-y)^2}{(y+x)(y-x)} = \frac{x-y}{x+y} \end{aligned}$$

$$\begin{aligned} \text{n) } \frac{x^2}{x^2-y^2} + \frac{y^2}{x^2-y^2} + \frac{-2xy}{-(y^2-x^2)} &= \frac{(x-y)^2}{(x+y)(x-y)} \\ &= \frac{x-y}{x+y} \end{aligned}$$

$$\begin{aligned} \text{o) } \frac{x+a}{x(a+b)+y(a+b)} - \frac{x-b}{x(a+b)+y(a+b)} &= \frac{x+a-x+b}{(x+y)(a+b)} \\ &= \frac{a+b}{(x+y)(a+b)} = \frac{1}{x+y} \end{aligned}$$

$$\begin{aligned} \text{p) } \frac{2x^2+1}{2x^2-5x-12} - \frac{4-x}{2x^2-5x-12} &= \frac{2x^2+1-4+x}{(2x+3)(x-4)} \\ &= \frac{x-3}{(2x+3)(x-4)} = \frac{x-3}{x-4} \end{aligned}$$

4. Perform the indicated operation. Simplify when possible. Assume the denominators are not zero.

a) $\frac{5}{6x^2} + \frac{4}{3x} \left(\frac{2x}{2x} \right)$

$$\frac{5 + 8x}{6x^2}$$

b) $\frac{5}{3x^2y^3} - \frac{1}{6xy^4}$

c) $\frac{4z}{z^2-36} - \frac{2}{z-6} \left(\frac{z+6}{z+6} \right)$

$$\begin{aligned} \frac{4z - 2z - 12}{(z-6)(z+6)} &= \frac{2(z-6)}{(z-6)(z+6)} \\ &= \frac{2}{z+6} \end{aligned}$$

d) $\frac{3x}{x^2-49} - \frac{3}{2x-14}$

e) $\frac{y}{y^2-9} + \frac{-3}{(3-y)} \left(\frac{y+3}{y+3} \right)$

$$\begin{aligned} \frac{y}{(y+3)(y-3)} + \frac{-3y-9}{(y+3)(y-3)} \\ \frac{-2y-9}{(y+3)(y-3)} \end{aligned}$$

f) $\frac{4z}{z-1} - 4$

g) $\frac{x+1}{x^2-x-6} - \frac{2}{x-3} \left(\frac{x+2}{x+2} \right)$

$$\frac{x+1-2x-4}{(x-3)(x+2)} = \frac{-x-3}{(x-3)(x+2)}$$

h) $\frac{4y}{y^2-5y} - \frac{3}{2y-10}$

i) $\frac{(z+4)}{(z+4)} \frac{1}{z-5} - \frac{z}{z^2-z-20}$

$$\frac{z+4-z}{(z-5)(z+4)} = \frac{4}{(z-5)(z+4)}$$

j) $\frac{4}{x^2-4} + \frac{1}{2-x} - \frac{1}{x+2}$

$$4. \text{ k) } \frac{4-y}{y-5} \cdot \frac{2}{y+5} + \frac{5y}{y^2-25} + \frac{-4}{5-y} \left(\frac{y+5}{y+5} \right)$$

$$l) \frac{z+2}{z^2+z-2} + \frac{3}{z^2-1}$$

$$\frac{2y-10+5y-4y-20}{(y-5)(y+5)(y-5)(y+5)} = \frac{3y-30}{(y-5)(y+5)}$$

$$\frac{3(y-10)}{(y-5)(y+5)}$$

$$m) \frac{3x+3}{x^2+5x+4} - \frac{x-3}{x^2+x-12}$$

$$n) \frac{x-1}{2x^2+3x+1} - \frac{x+1}{2x^2-x-1}$$

$$\frac{3(x+1)}{(x+4)(x+1)} - \frac{(x-3)}{(x+4)(x-3)}$$

$$\frac{3-1}{x+4} = \frac{2}{x+4}$$

$$o) \frac{x}{2x^2+x-1} + \frac{3}{3x^2+2x-1}$$

$$p) \frac{5}{2x^3} - \frac{3x-9}{x^2-6x+9} + \frac{12x}{4x^2-12x}$$

$$\frac{x}{(2x-1)(x+1)} + \frac{3}{(3x-1)(x+1)}$$

$$\frac{x(3x-1) + 3(2x-1)}{(2x-1)(x+1)(3x-1)} = \frac{3x^2+5x-3}{(2x-1)(x+1)(3x-1)}$$

$$q) \frac{y-5}{(x^2+5x)+(xy+5y)} + \frac{1}{x+y} - \frac{2}{x+5}$$

$$r) \frac{2z+11}{z^2+z-6} - \frac{2}{z+3} + \frac{3}{2-z}$$

$$\frac{y-5}{x(x+5)+y(x+5)} + \frac{1}{(x+5)(x+4)} - \frac{2}{x+5}$$

$$\frac{y-5+x-5-2(x+4)}{(x+5)(x+4)(x+5)} = \frac{-4-x}{(x+5)(x+4)(x+5)} = \frac{-1}{(x+5)(x+4)}$$

$$s) \frac{x+1}{x^3+x^2-9x-9} - \frac{x-3}{x^3-3x^2-x+3}$$

$$t) \frac{a}{(b-a)(c-a)} - \frac{b}{(b-c)(a-b)} - \frac{c}{(a-c)(c-b)}$$

$$\frac{x^2(x+1)-9(x+1)}{(x+1)(x+3)(x-3)} - \frac{x(x-3)}{(x-3)(x+1)(x-1)}$$

$$\frac{(x+1)(x-1) - (x-3)(x+3)}{(x+1)(x-1)(x+3)(x-3)}$$

$$\frac{x^2-1-x^2+9}{(x+1)(x-1)(x+3)(x-3)} = \frac{8}{(x+1)(x-1)(x+3)(x-3)}$$

$$\frac{x^2-1-x^2+9}{(x+1)(x-1)(x+3)(x-3)} = \frac{8}{(x+1)(x-1)(x+3)(x-3)}$$

5. A stick of length $5/(x+3)$ cm is cut into two pieces. If one piece is $2/(x-3)$ cm, find the length of the other piece.
6. The length of a rectangle is $2/(x-4)$ m, and the width is $3/x$ m. Find the perimeter of the rectangle.

$$\frac{5}{x+3} - \frac{2}{x-3}$$

$$\frac{5(x-3) - 2(x+3)}{(x+3)(x-3)} = \frac{5x-15-2x-6}{(x+3)(x-3)}$$

$$= \frac{3x-21}{(x+3)(x-3)} = \frac{3(x-7)}{(x+3)(x-3)} \text{ cm.}$$

7. Cold water can fill a bathtub in t minutes, and hot water can fill the same bathtub in $t+2$ minutes. If the hot and cold water are both running, how much of the bathtub is filled in 4 minutes?

$$\frac{4}{t} + \frac{4}{t+2}$$

$$\frac{4(t+2) + 4t}{t(t+2)} = \frac{8t+8}{t(t+2)} = \frac{8(t+1)}{t(t+2)}$$

9. George drove for 100 km at a constant speed, then increased his speed by 10 km/h and drove 200 km more. Write a rational expression for the total time.

S	T	D	
x	$\frac{100}{x}$	100	
$x+10$	$\frac{200}{x+10}$	200	

$$\frac{100}{x} + \frac{200}{x+10} = \frac{100(x+10) + 200x}{x(x+10)}$$

8. Sam averages x hours to paint a room, and Bill averages $x+1$ hours to paint the same rooms. Write a rational expression for the number of rooms painted while Sam and Bill work together on an 8 hour shift.

10. Write the infinite sum, $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots$, as a rational number.

11. The sum of a rational expression and $1/(x+3)$ is $3x/(2x^2+5x-3)$. Determine the rational expression.

$$\frac{3x}{(2x-1)(x+3)} - \frac{1}{x+3}$$

$$\frac{3x - 1(2x-1)}{(2x-1)(x+3)} = \frac{x+1}{(2x-1)(x+3)}$$

12. The difference formed by a rational expression minus $1/(x+3)$ is $-x/(3x^2+7x-6)$. Determine the rational expression.

2.4 Exercise Set

1. Simplify.

a) $\frac{5}{x} - \frac{3}{x^3} \div \frac{2}{x}$

$$\frac{\cancel{2x} \cdot 5}{\cancel{2x} \cdot x} - \left(\frac{3}{x^3} \cdot \frac{x}{2} \right)$$

$$\frac{10x-3}{2x^2}$$

b) $\frac{4}{2x^3} - \frac{5x+10}{x^8} \div \frac{x+2}{x^3}$

d) $\left(\frac{x-7}{x^2-16} - \frac{x-1}{16-x^2} \right) \left(\frac{x^2-16}{2} \right)$

c) $\frac{2}{x} + \frac{x^2-y^2}{4x+4y} \cdot \frac{12x^2}{3y-3x}$

$$\frac{2}{x} + \frac{(x+y)(x-y) \cdot \cancel{12}x^2}{\cancel{4}(x+y) \cdot \cancel{3}(y-x)}$$

$$\frac{2}{x} - x^2 \left(\frac{x}{x} \right) = \frac{2-x^3}{x}$$

e) $\left(\frac{y+3}{y-5} + \frac{y-2}{y+4} \right) (y^2-y-20)$

$$\frac{(y+3)(y+4) + (y-2)(y-5) \cdot (y-8)(y+4)}{(y-8)(y+4)}$$

$$y^2+7y+12 + y^2-7y+10$$

$$2y^2+22 = 2(y^2+11)$$

g) $\frac{2}{b^2} + \frac{6ab}{4ab+4b^2} \div \frac{7a-7b}{a^2-b^2}$

$$\frac{2}{b^2} + \frac{\cancel{6a}b}{\cancel{4}b(a+b)} \cdot \frac{(a+b)(a-b)}{\cancel{7}(a-b)}$$

$$\frac{2}{b^2} + \frac{3a}{14} = \frac{2(14) + 3ab^2}{14b^2}$$

f) $\left(\frac{x}{x^2-16} - \frac{2}{3x+12} \right) \left(\frac{x-4}{6} \right)$

h) $\frac{x^3}{3} - \frac{2x^2+xy}{xy} \cdot \frac{y}{10x+5y}$

$$x^2+2x+1 \left(\frac{x+6}{x+2} - \frac{4}{x} \right) \div \frac{x^2-16}{x^2-4}$$

$$\frac{x(x+6)-4(x+2)}{x(x+2)} \cdot \frac{(x+2)(x-2)}{(x+4)(x-4)}$$

$$\frac{(x+4)(x-2)}{x} \cdot \frac{(x-2)}{(x+4)(x-4)} = \frac{(x-2)^2}{x(x-4)}$$

j) $\frac{2x^2}{x-1} - \frac{2x^2-7x+3}{x-3} \cdot \frac{x+2}{x-1}$

2. Simplify.

$$a) \frac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} \times \frac{6}{5} = \frac{4}{5}$$

$$b) \frac{\frac{x}{y^3}}{\frac{x^2}{y}}$$

$$c) \frac{\frac{8x^4y^3}{3x}}{\frac{4xy^4}{y^2}} = \frac{8x^4y^3}{3x} \cdot \frac{y^2}{4xy^4} = \frac{2x^2y}{3}$$

$$d) \frac{\frac{x+2}{5}}{\frac{x-3}{x}}$$

$$e) \frac{\frac{1}{x} - 2\left(\frac{x}{x}\right)}{\frac{5}{x} + 1\left(\frac{x}{x}\right)} = \frac{1-2x}{5+x}$$

$$f) \frac{1 + \frac{8}{y}}{y + 3 - \frac{40}{y}}$$

$$g) \frac{1 - \frac{4}{z} + \frac{4}{z^2}}{\frac{1}{z^2} - \frac{2}{z^3}} = \frac{(z-2)^2}{z^2 - 4z + 4} \cdot \frac{z^3}{z^2} = z(z-2)$$

$$h) \frac{\frac{4}{x^2} - \frac{12}{xy} + \frac{9}{y^2}}{\frac{4}{x^2} - \frac{9}{y^2}}$$

$$i) \frac{\frac{3}{x-2} + \frac{2}{x+2}}{\frac{4}{x+2} - \frac{5}{x-2}} = \frac{\frac{3(x+2) + 2(x-2)}{(x+2)(x-2)}}{\frac{4(x-2) - 5(x+2)}{(x+2)(x-2)}}$$

$$j) \frac{\frac{1}{x-1} - 2}{\frac{3}{x-1} + 4}$$

$$\frac{3x+6+2x-4}{4x-8-5x-10} = \frac{5x+2}{-x-18}$$

$$2. \quad k) \quad \frac{\frac{2}{x-3} + \frac{4}{x}}{\frac{3}{x-2} + \frac{1}{x}} = \frac{2x+4(x-3)}{3x+(x-2)}$$

$$\frac{6x-12}{x(x-3)} \cdot \frac{x(x-2)}{4x-2} = \frac{3x+(x-2)}{x(x-2)}$$

$$\frac{3(x-2)}{x(x-3)} \cdot \frac{x(x-2)}{2(x-1)} = \frac{3(x-2)^2}{(x-3)(2x-1)}$$

$$m) \quad \frac{\frac{z}{3} - \frac{z-1}{9-z}}{\frac{z}{6} - \frac{2-z}{z-9}} = \frac{z(9-z) - 3(z-1)}{z(z-9) - 6(2-z)}$$

$$\frac{9z-z^2-3z+3}{3(9-z)} = \frac{z(9-z) - 6(2-z)}{z(z-9) - 6(2-z)}$$

$$\frac{z^2-9z-12+6z}{6(z-9)} = -\frac{(z^2-6z-3) \cdot 6(z-9)}{z^2-3z-12}$$

$$= \frac{2(z^2-6z-3)}{z^2-3z-12}$$

$$o) \quad \frac{\frac{1}{x^2+x} - \frac{1}{xy+y}}{\frac{1}{xy+y} - \frac{1}{x^2+x}} = \frac{\frac{1}{x(x+1)} - \frac{1}{y(x+1)}}{\frac{1}{y(x+1)} - \frac{1}{x(x+1)}}$$

$$\frac{\frac{y-x}{xy(x+1)}}{\frac{x-y}{xy(x+1)}} = \frac{y-x}{x-y} = -1$$

$$l) \quad \frac{3 - \frac{4}{y-1}}{5 - \frac{3}{1-y}}$$

$$n) \quad \frac{x + \frac{2}{x+3}}{x-5 + \frac{12}{x+3}}$$

$$p) \quad \frac{\frac{2}{y^2-1} + \frac{3}{x-xy}}{\frac{3}{xy-x} - \frac{2}{y^2-1}}$$

$$q) \quad \frac{\frac{x}{x^2-1} - \frac{3x+3}{1-x}}{\frac{2x-1}{x-1} + \frac{x}{1-x}} = \frac{\frac{x}{(x+1)(x-1)} + \frac{3(x+1)}{x-1}}{\frac{2x-1}{x-1} - \frac{x}{x-1}}$$

$$\frac{x+3(x+1)(x+1)}{(x+1)(x-1)} = \frac{x+3x^2+6x+3}{(x+1)(x-1)} \cdot \frac{(x-1)}{x-1}$$

$$\frac{2x-1-x}{x-1} = \frac{3x^2+7x+3}{(x+1)(x-1)}$$

$$r) \quad \frac{x + \frac{5}{1 + \frac{x}{2}}}{x - \frac{4}{2 + \frac{x}{3}}}$$

$$s) \quad \frac{\frac{x}{x+1} + 3}{\frac{x}{x+1} + \frac{3}{x}} = \frac{x+3(x+1)}{x+1} \cdot \frac{x}{x+1}$$

$$\frac{4x+3}{x+1} = \frac{x}{x+1} + 3 \cdot \frac{x+1}{x}$$

$$\frac{x^2+3(x+1)^2}{x(x+1)} = \frac{(4x+3) \cdot x}{x^2+3x^2+6x+3} = \frac{4x^2+3x}{4x^2+6x+3}$$

3. Simplify each complex fraction.

$$a) \frac{x^{-2} + x}{x} = \frac{\frac{1}{x^2} + x}{1} = \frac{1 + x \cdot x^2}{x^2}$$

$$b) \frac{x^{-2} - 3x^{-3}}{3x^{-2} - 9x^{-3}}$$

$$\frac{1+x^3}{x^2} \cdot \frac{1}{x} = \frac{1+x^3}{x^3}$$

$$c) (x^{-1} + y^{-1})^{-1} \\ \left(\frac{1}{x} + \frac{1}{y}\right)^{-1} = \left(\frac{y+x}{xy}\right)^{-1} \\ = \frac{xy}{x+y}$$

$$d) (x^{-1} - y^{-1})^{-2}$$

4. Simplify the formula for the total resistance of an electrical circuit.

$$T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

5. Solve the formula for R , then for r .

$$i = \frac{E}{R + \frac{r}{2}}$$

6. The relationship between the focal length, f , of a camera lens, the distance, d_o , of the object from a lens, and the distance to the image, d_i , is given by $f^{-1} = d_i^{-1} + d_o^{-1}$. Solve the formula for f .

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$\frac{1}{f} = \frac{d_o + d_i}{d_i d_o}$$

$$f = \frac{d_i d_o}{d_o + d_i}$$

$$7. \text{ Simplify } \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}}$$

2.5 Exercise Set

1. Fill in the blanks to make the statement true.

a) Rejected solutions that make the denominator zero are called extra-
aneous solutions.b) To eliminate an equation of fractions, both sides of the fraction must be multiplied by the LCD of the equation.c) If you multiply both sides of an equation that has a variable in the denominator, you must check the solution.d) To eliminate the equation $\frac{1}{x} + \frac{3}{2x} = 7$ of fractions, multiply both sides by $2x$.e) To eliminate the equation $\frac{1}{x+1} + \frac{1}{x-1} = \frac{1}{x}$ of fractions, multiply both sides by $x(x+1)(x-1)$.

2. Simplify the expression or solve the equation.

a) $\frac{3}{4}x - \frac{2}{3}x$ $\frac{9x-8x}{12} = \frac{x}{12}$

b) $\frac{3}{4}x - \frac{2}{3}x = 2$

c) $\frac{y}{2} - \frac{4y}{3} + 5$

d) $\frac{y}{2} - \frac{4y}{3} + 5 = 0$

$$\frac{3y-8y+30}{6} = \frac{-5y+30}{6}$$

e) $\frac{3(z-1)}{2} - z + 2$

f) $\frac{3(z-1)}{2} = z - 2$

$$\frac{3z-3-2z+4}{2} = \frac{z+1}{2}$$

g) $\frac{x}{x-2} - \frac{2}{x-2} - 2$

h) $\frac{x}{x-2} = \frac{2}{x-2} + 2$

$$\begin{aligned} \frac{x-2-2(x-2)}{x-2} &= \frac{x-2-2x+4}{x-2} \\ &= \frac{-x+2}{x-2} = -1 \end{aligned}$$

3. Determine what values could not possibly be solutions to the following equations. Do not solve the equation.

a) $\frac{x}{3} - \frac{2}{x-1} = 7$

b) $\frac{x-1}{2} + \frac{x^2-1}{3} = 2$

$x \neq \pm 1$

c) $\frac{x^2+9}{1} + \frac{x+1}{1} = 4$

d) $\frac{x^2-4}{1} - \frac{x^2-x-6}{3} = 0$

$x \neq \pm 2, 3$

e) $\frac{x-6}{2} - \frac{x^2-9}{3} + \frac{x^2+1}{1} = 2$

$x \neq \pm 3$

g) $\frac{x^2-y^2}{3} + \frac{3x+6y}{2} = 1$

$3x+6y \neq 0$
 $3x \neq -6y$
 $x \neq -2y$

4. Solve the equation.

a) $\left[\frac{2}{x} + \frac{3}{x} = 5 \right]$

$3x+2x=30$
 $5x=30$
 $x=6$

b) $\frac{3}{y} + 5 = \frac{4}{3y}$

c) $\left[\frac{x}{4} - x - \frac{2}{3} = 0 \right]$

$x-4x-6=0$
 $-3x-6=0$
 $-3x=6$
 $x=-2$

d) $\frac{2}{y} + \frac{4}{5y} = \frac{12}{y}$

e) $\left[\frac{4}{3(x+1)} = x+1 \right]$

$3(x+1)=4(x+1)$
 $3x+3=4x+4$
 $-1=x$

h) $\frac{5}{3x} - \frac{x-5}{7} = 3$

$\left[\frac{x-4}{3} - \frac{x-2}{2} = -\frac{6}{5} \right]$

$2(x-4) - 3(x-2) = -5$
 $2x-8-3x+6=-5$
 $-x=-3$
 $x=3$

j) $\frac{5}{4x+1} = \frac{3}{8x+2} + 1$

i) $\left[\frac{x+2}{4} - \frac{x-1}{2} = \frac{3}{2} \right]$

$3(x+2) - 2(x-1) = 6$
 $3x+6-2x+2=6$
 $x+8=6$
 $x=-2$

$-3x=-4$
 $x=4/3$

5. Solve. Check your solutions.

a) $\left[\frac{5}{3x} - \frac{1}{9} = \frac{4}{x}\right] 9x$

$$5(3) - x = 4(9)$$

$$15 - x = 36$$

$$-x = 21$$

$$x = -21$$

b) $\frac{7}{x+3} = \frac{5}{x-9}$

c) $\left[\frac{y}{y+3} - 2 = \frac{-3}{y+3}\right] (y+3)$

$$y - 2(y+3) = -3$$

$$y - 2y - 6 = -3$$

$$-y = 3$$

$$y = -3 \quad \text{no solution } \emptyset$$

e) $\left[\frac{2}{z+5} + \frac{20}{z^2-25} = \frac{+3}{z-5}\right] (z+5)(z-5)$

$$2(z-5) + 20 = 3(z+5)$$

$$2z - 10 + 20 = 3z + 15$$

$$-z = z \quad \text{no solution } \emptyset$$

g) $\left[\frac{2y}{y^2-1} = \frac{2}{y+1} + \frac{-1}{y-1}\right] (y+1)(y-1)$

$$2y = 2(y-1) - 1(y+1)$$

$$2y = 2y - 2 - y - 1$$

$$y = -3$$

i) $\left[\frac{2}{x} - \frac{x}{5x-12} = 0\right] x(5x-12)$

$$2(5x-12) - x^2 = 0$$

$$10x - 24 - x^2 = 0$$

$$x^2 - 10x + 24 = 0 \quad (x-6)(x-4) = 0$$

k) $\left[\frac{y}{2y+2} + \frac{2y-16}{4y+4} = \frac{2y-3}{y+1}\right] 4(y+1)$

$$2y + 2y - 16 = 4(2y-3)$$

$$4y - 16 = 8y - 12$$

$$-4 = 4y \quad y = -1 \quad \text{no solution } \emptyset$$

d) $\frac{1}{2x} + \frac{4}{x} = \frac{9}{2x}$

f) $\frac{3}{x-1} + \frac{1}{2x-2} = \frac{7}{4}$

h) $\frac{z}{2z+2} + \frac{2z}{4z+4} = \frac{2z-3}{z+1}$

j) $\frac{y^2+3}{y-1} = \frac{4}{y-1}$

l) $\frac{5}{4z-2} - \frac{1}{1-2z} = \frac{7}{3z+6}$

$$5. \quad m) \left[\frac{2x+3}{x-1} - \frac{2}{x+3} = \frac{5-6x}{x^2+2x-3} \right] (x-1)(x+3) \quad n) \frac{x+1}{x+3} + \frac{x-3}{x-2} = \frac{x^2-11x}{x^2+x-6}$$

$$(2x+3)(x+3) - 2(x-1) = 5-6x$$

$$2x^2+6x+3x+9-2x+2 = 5-6x$$

$$2x^2+13x+6=0$$

$$(2x+1)(x+6)=0 \quad x = -1/2, -6$$

$$o) \left[\frac{3-2x}{x+1} - \frac{10}{x^2-1} = \frac{2x+3}{1-x} \right] (x+1)(x-1) \quad p) \frac{3}{z-3} + \frac{5}{z+2} = \frac{5z}{z^2-z-6}$$

$$(3-2x)(x-1) - 10 = (x+1)(-2x-3)$$

$$3x-3-2x^2+2x-10 = -2x^2-3x-2x-3$$

$$10x = 10$$

$$x = 1 \quad \text{no solution } \phi$$

$$q) \left[\frac{1}{(x-1)^2} - 3 = \frac{-2}{1-x} \right] (x-1)(x-1) \quad r) \frac{x^2}{x^2-x-2} = \frac{2x}{x^2+x-6}$$

$$1 - 3(x-1)^2 = -2(x-1)$$

$$1 - 3x^2 + 6x - 3 = -2x + 2$$

$$0 = 3x^2 - 8x + 4$$

$$0 = (3x-2)(x-2)$$

$$x = 2/3, 2$$

$$(x+2)(x-1)(x+1)$$

$$s) \left[\frac{x}{x^2+x-2} + \frac{x}{x^2-1} = \frac{x}{x^2+3x+2} \right] (x+2)(x-1)(x+1)(x+2)(x-1)(x+1)$$

$$x(x+1) + x(x+2) = x(x-1)$$

$$x^2+x+x^2+2x = x^2-x$$

$$x^2+4x=0$$

$$x(x+4)=0$$

$$x=0, -4$$

$$u) \left[\frac{x}{x-3} + \frac{1}{x-2} - \frac{1}{x+2} = \frac{x-12}{x^3-3x^2-4x+12} \right] (x-3)(x-2)(x+2) \quad v) \frac{3y-7}{y^2-5y+6} + \frac{2y+8}{9-y^2} - \frac{y+2}{y^2+y-6} = 0$$

$$(x-3)(x-2)(x+2)$$

$$x(x^2-4) + (x-3)(x+2) - (x-3)(x-2) = x-12$$

$$x^3-4x+x^2-x-6-x^2+5x-6 = x-12$$

$$x^3-x=0 \quad x(x+1)(x-1)=0$$

$$x(x^2-1)=0 \quad x=0, \pm 1$$

6. Solve each formula for the indicated variable.

a) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_1

$$\frac{1}{R} - \frac{1}{R_2} = \frac{1}{R_1}$$

$$\frac{R_2 - R}{RR_2} = \frac{1}{R_1} \quad R_1 = \frac{RR_2}{R_2 - R}$$

c) $d = \frac{2s}{n(a+l)}$ for l

$$n(a+l) = \frac{2s}{d} \quad a+l = \frac{2s}{nd}$$

$$l = \frac{2s}{nd} - a$$

e) $4x + \frac{3}{y} = \frac{2}{z}$ for y

$$\frac{3}{y} = \frac{2}{z} - 4x$$

$$\frac{3}{y} = \frac{2 - 4xz}{z}$$

g) $A = \frac{1}{2}h(a+b)$ for a

$$\frac{2A}{h} = a + b$$

$$\frac{2A}{h} - b = a$$

i) $S = \frac{1}{2}gt^2 + v_0t + s_0$ for g

$$S - v_0t - s_0 = \frac{1}{2}gt^2$$

$$\frac{2(S - v_0t - s_0)}{t^2} = g$$

k) $-3x - \frac{4}{y} = \frac{6}{z}$ for y

$$-3x - \frac{4}{y} = \frac{6}{z}$$

$$\frac{-3xz - 4}{z} = \frac{6}{y}$$

b) $I = \frac{E}{R+r}$ for r

d) $\frac{2}{x} = \frac{1}{y} - \frac{1}{z}$ for z

f) $\frac{a}{1+bc} = d$ for c

h) $F = \frac{9}{5}C + 32$ for C

j) $S = 2LH + 2LW + 2HW$ for L

l) $\frac{5}{x} + \frac{2}{y} + \frac{3}{z} = 1$ for z

2.6 Exercise Set

1. Unknown number problems.

- a) The sum of a number and its reciprocal is $\frac{13}{6}$. Find the number.

$$\left[x + \frac{1}{x} = \frac{13}{6} \right] 6x$$

$$6x^2 + 6 = 13x$$

$$6x^2 - 13x + 6 = 0$$

$$(3x - 2)(2x - 3) = 0$$

$$x = \left[\frac{2}{3}, \frac{3}{2} \right]$$

- c) Find two consecutive integers whose reciprocals add to $\frac{7}{12}$.

- b) The sum of an integer and its reciprocal is $\frac{65}{8}$. Find the integer.

$$\left[\frac{1}{x} + \frac{1}{x+1} = \frac{7}{12} \right] 12x(x+1)$$

$$12(x+1) + 12x = 7x(x+1)$$

$$12x + 12 + 12x = 7x^2 + 7x$$

$$0 = 7x^2 - 17x - 12$$

$$0 = (7x + 4)(x - 3)$$

$$x = -\frac{4}{7}, 3$$

- e) Find two consecutive even integers whose reciprocals add to $\frac{7}{24}$.

- d) Find two consecutive odd integers whose reciprocals add to $\frac{8}{15}$.

$$\left[\frac{1}{x} + \frac{1}{x+2} = \frac{7}{24} \right] 24x(x+2)$$

$$24(x+2) + 24x = 7x(x+2)$$

$$24x + 48 + 24x = 7x^2 + 14x$$

$$0 = 7x^2 - 34x - 48$$

$$0 = (7x + 8)(x - 6)$$

- g) A number added to the product of 6 and the reciprocal of the number is -5 . Find the number.

- f) The quotient of a number and 5 minus $\frac{3}{2}$ is the quotient of the number and 2. Find the number.

$$\left[\frac{6}{x} + x = -5 \right] x$$

$$6 + x^2 = -5x$$

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0$$

$$x = -3, -2$$

2. Work problems.

- a) It would take Sue 4 hours to paint a large room, and it would take Bert 5 hours to paint the same room. If they work together, how long would it take them to complete the job?

$$\left[\frac{x}{4} + \frac{x}{5} = 1 \right] 20$$

$$5x + 4x = 20$$

$$9x = 20$$

$$x = 20/9 \text{ hrs.}$$

- b) One worker takes 20 hours to do a job. A second worker can do the same job in 15 hours. When a third worker is added, it takes the three of them 6 hours to do the job. How long would it take the third worker to do the job?

- c) Jane works twice as fast as her daughter Anna. If it takes 15 minutes to clean the kitchen together, how long would it take Anna to clean the kitchen by herself?

$$\left[\frac{15}{x} + \frac{15}{2x} = 1 \right] 2x$$

$$30 + 15 = 2x$$

$$45 = 2x$$

$$x = 45/2 = 22.5 \text{ min}$$

∴ Anna takes 45 min.

- d) Ken takes 3 hours longer to assemble a motor than Hans. When working together, it takes them 2 hours to assemble the motor. How long would it take Ken to do the job alone?

- e) A cold water tap can fill a tub in 6 minutes, and a hot water tap can fill the tub in 8 minutes. A drain can empty a full tub in 10 minutes. If both taps are on and the drain is open, how long will it take to fill the tub?

$$\left[\frac{x}{6} + \frac{x}{8} - \frac{x}{10} = 1 \right] 240$$

$$40x + 30x - 24x = 240$$

$$46x = 240$$

$$x = 240/46 = 120/23 \text{ min.}$$

$$= 5 \text{ hrs. } 13 \text{ min.}$$

- f) A brick layer can build a wall in 6 hours, and his apprentice would need 16 hours to build the same wall. When they work together, the apprentice works 5 hours longer than the brick layer. How many hours does each work?

3. Distance Problems

- a) A boat travels 40 km downstream in the same time it takes to travel 30 km upstream. If the current flows at 6 km/h, what is the speed of the boat in still water?

	S	T	D
U	$B-6$	$\frac{30}{B-6}$	30
D	$B+6$	$\frac{40}{B+6}$	40

$$\left[\frac{30}{B-6} = \frac{40}{B+6} \right] (B-6)(B+6) \quad 420 = 10B$$

$$30(B+6) = 40(B-6) \quad 42 \text{ km/h} = B$$

$$30B + 180 = 40B - 240$$

- c) A woman drives to work at an average speed of 50 mph. The average speed of the return trip home is 30 mph. What is the average speed for the round trip?

- b) The speed of a boat in stillwater is 10 mph. The boat travels 24 miles upstream and back downstream in a total of 5 hours. What is the speed of the current?

- d) On a 100 km round trip, Jessica averages 40 km/h to her destination and 60 km/h returning. What is the average speed for the entire trip?

- e) Alejandro decides to go to the store which is 8 km from his house. He first ran at a rate of 7 km/h, then walked the rest of the trip at a speed of 3 km/h. If the total trip took 2 hours, how many kilometers did he walk?

- f) Driver A and B have two different delivery routes. Driver A's route is 80 km, and driver B's route is 100 km. Driver B travels 10 km/h faster than driver A and finishes 10 minutes earlier. What are the speeds of each driver?

	S	T	D
run	7	x	$7x$
walk	3	$2-x$	$3(2-x)$

$$7x + 3(2-x) = 8$$

$$7x + 6 - 3x = 8$$

$$4x = 2$$

$$x = 1/2$$

$$\text{walk} = 3(2 - 0.5) = 4.5 \text{ km}$$

2.7

Chapter Review

Section 2.1

1. Simplify the rational expressions. State the restrictions.

a) $\frac{x^2 + x - 6}{2 - x}$ $\frac{(x+3)(x-2)}{2-x}$

$-(x+3)$

$x \neq 2$

b) $\frac{3x^2 - 13x - 30}{15x^2 + 28x + 5}$

c) $\frac{6x^2 + x - 1}{8x^2 - 2x - 3}$ $\frac{(3x-1)(2x+1)}{(4x-3)(2x+1)}$

$\frac{3x-1}{4x-3}$ $x \neq 3/4, -1/2$

d) $\frac{2x^3 - 2x^2 - 12x}{3x^2 - 6x}$

e) $\frac{4x^2 - 8x + 3}{4x^2 - 6x + 2xy - 3y}$ $\frac{(2x-1)(2x-3)}{(2x+4)(2x-3)}$

$2x(x-3) \cdot y(2x-3)$

$\frac{2x-1}{2x+4}$ $x \neq -\frac{4}{2}, \frac{3}{2}$

g) $\frac{2x^2 + 5xy + 3y^2}{4x^2 + 12xy + 9y^2}$

$\frac{(2x+3y)(x+y)}{(2x+3y)(2x+3y)}$

$\frac{x+y}{2x+3y}$

$x+y$

$2x+3y$

$x \neq -\frac{3y}{2}$

h) $\frac{4xy - 2x^2}{ax + bx - 2ay - 2by}$

Section 2.2

2. Multiply. Simplify answer if possible.

a) $\frac{x-3}{2x^2-5x-3} \cdot \frac{8(2x+1)}{16x+8}$

$\frac{(2x+1)(x-3)}{(2x+1)(x-3)}$

$= 8$

b) $\frac{2x^2 - 9x - 5}{2x + 1} \cdot \frac{2}{5 - x}$

c) $\frac{x^2 + 7x + 12}{x^2 + 2x - 8} \cdot \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$

$\frac{(x+3)(x+4)}{(x+4)(x-2)} \cdot \frac{(x-2)(x-1)}{(x+3)(x-1)}$

$= 1$

d) $\frac{x^2 + x - 2}{x^2 + 3x + 2} \cdot \frac{x^2 - 4x - 5}{x^2 - 3x + 2}$

3. Divide. Simplify answer if possible.

a) $\frac{2x^2 - 5x - 12}{3x^2 - 11x - 4} \div \frac{2x^2 - 7x - 15}{3x^2 - 14x - 5}$

$$\frac{(2x+3)(x-4)}{(3x+1)(x-4)} \cdot \frac{(3x+1)(x-5)}{(2x+3)(x-5)} = 1$$

b) $\frac{6x^2 - xy - 2y^2}{8x^2y + 4xy^2} \div \frac{4xy^2 - 6x^2y}{8x^2y^2}$

c) $\left(\frac{2a^2 - 5ab - 3b^2}{3xya - xyb} \div \frac{a - 3b}{x^2y} \right) \div \frac{3a - b}{2a + b}$

$$\frac{(2a+b)(a-3b)}{xy(3a-b)} \cdot \frac{x^2y}{a-3b} \cdot \frac{2a+b}{3a-b} = \frac{x(2a+b)^2}{(3a-b)^2}$$

d) $\frac{x^2 + 7xy + 12y^2}{x^2 + 2xy - 8y^2} \div \frac{x^2 + 2xy - 3y^2}{x^2 - 3xy + 2y^2} \cdot \frac{9 - x^2}{x - 3}$

Section 2.3

4. Perform the indicated operation. Simplify if possible.

a) $\frac{2}{x} - \frac{3}{x+1} + \frac{1}{x-1}$

$$\frac{2(x+1)(x-1) - 3x(x-1) + x(x+1)}{x(x+1)(x-1)} = \frac{2x^2 - 2 - 3x^2 + 3x + x^2 + x}{x(x+1)(x-1)} = \frac{4x-2}{x(x+1)(x-1)} = \frac{2(2x-1)}{x(x+1)(x-1)}$$

b) $\frac{x-1}{2x^2 + 3x + 1} - \frac{x+1}{2x^2 - x - 1}$

c) $\frac{x-3}{x+3} + \frac{x+3}{3-x} + \frac{36}{x^2-9}$

d) $\frac{a+b}{ax+ay} + \frac{a+b}{bx+by}$

$$\frac{(x-3)(x-3) - (x+3)(x+3) + 36}{(x+3)(x-3)} = \frac{x^2 - 6x + 9 - x^2 - 6x - 9 + 36}{(x+3)(x-3)} = \frac{-12x + 36}{(x+3)(x-3)} = \frac{-12(x-3)}{(x+3)(x-3)} = \frac{-12}{x+3}$$

e) $\frac{2-x}{x^2+2x} + \frac{x}{x^2+4x+4}$

f) $\frac{x+1}{x^2-3x} + \frac{x-2}{x^2-6x+9}$

$$\frac{(2-x)(x+2) + x^2}{x(x+2)(x+2)} = \frac{2x+4-x^2-2x+x^2}{x(x+2)^2} = \frac{4}{x(x+2)^2}$$

g) $\frac{1}{x} + \frac{x}{2x+4} - \frac{2}{x^2+2x}$

h) $\frac{2x+8}{x^2+5x+6} - \frac{x-1}{x^2+3x+2} - \frac{x+5}{x^2+4x+3}$

$$\frac{2(x+2) + x^2 - 2(2)}{2x(x+2)} = \frac{2x+4+x^2-4}{2x(x+2)} = \frac{x^2+2x}{2x(x+2)} = \frac{x}{2}$$

Section 2.4

5. Simplify.

a) $\left(\frac{x+6}{x+2} - \frac{4}{x}\right)\left(\frac{x^2-4}{x^2-16}\right)$

$$x(x+6) - 4(x+2)$$

$$x^2 + 6x - 4x - 8$$

$$\frac{(x+4)(x-2) \cdot (x+2)(x-2)}{x(x+2) \cdot (x+4)(x-4)} = \frac{(x-2)^2}{x(x-4)}$$

c) $\frac{\frac{x-3}{y^2}}{\frac{3}{y^2} - \frac{2}{x}}$

$$\frac{x^2-3}{y^2} \cdot \frac{xy^2}{3x-2y^2}$$

$$\frac{x(xy^2-3)}{3x-2y^2}$$

b) $\left(1 - \frac{4}{x^2}\right) \div \left(\frac{2}{x^2} - \frac{1}{x}\right)$

d) $\frac{3x - \frac{12}{x}}{x - \frac{2}{3}} \cdot \frac{3}{x+1}$

e) $\left(x + \frac{2x}{x-1}\right) \div \left(x - \frac{2x}{x-1}\right)$

$$\frac{x(x-1)+2x}{x-1} \cdot \frac{x-1}{x(x-1)-2x} \rightarrow \frac{x^2-x+2x}{x^2-x-2x}$$

$$\frac{x^2+x}{x^2-3x} = \frac{x(x+1)}{x(x-3)} = \frac{x+1}{x-3}$$

g) $\frac{3 + \frac{x}{x+1}}{\frac{x}{x+1} + \frac{3x+3}{x}}$

$$\frac{3(x+1)+x}{x+1} \cdot \frac{x(x+1)}{x^2+(x+1)(3x+3)}$$

$$\frac{(3x+3+x)x}{x^2+3x^2+6x+3} = \frac{4x^2+3x}{4x^2+6x+3}$$

Section 2.5 x^2+3x^2+6x+3 $4x^2+6x+3$

6. Solve.

a) $\left[\frac{x}{4} - \frac{x+3}{6} = \frac{x-3}{3}\right]^{12}$

$$3x - 2(x+3) = 4(x-3)$$

$$3x - 2x - 6 = 4x - 12$$

$$x - 6 = 4x - 12 \quad 6 = 3x \quad x = 2$$

d) $\left[\frac{4}{2x-1} + \frac{2}{x+3} = \frac{-6}{2x^2+5x-3}\right] (2x-1)(x+3)$

$$4(x+3) + 2(2x-1) = -6 \quad (2x-1)(x+3)$$

$$4x+12+4x-2 = -6 \quad 8x = -16$$

$$8x+10 = -6 \quad x = -2$$

e) $\left[\frac{5x+1}{3x+3} = \frac{5x-5}{5x+5} + \frac{3x-1}{x+1}\right] 15(x+1)$

$$5(5x+1) = 3(5x-5) + 15(3x-1)$$

$$25x+5 = 15x-15+45x-15$$

$$25x+5 = 60x-30 \quad 35 = 35x \quad x = 1$$

b) $\frac{7}{x-5} + 2 = \frac{x+2}{x-5}$

d) $\frac{2}{x+1} = \frac{3}{1-x} + \frac{5}{x}$

f) $\frac{x-4}{x^2-5x} = \frac{2}{x^2-25}$

7. Solve for x .

a) $y = \frac{3x-4}{2x-3}$

b) $(3x-2)(2y-1) = z$

$$y(2x-3) = 3x-4$$

$$2xy - 3y = 3x - 4$$

$$2xy - 3x = 3y - 4$$

$$x(2y-3) = 3y-4$$

$$x = \frac{3y-4}{2y-3}$$

Section 2.6

8. The sum of two numbers is 38. If the larger number is divided by the smaller number, the quotient is 6 and the remainder is 3. Find the two numbers.

larger x
smaller $38-x$.

$$\left[\frac{x}{38-x} = 6 + \frac{3}{38-x} \right] 38-x$$

$$x = 6(38-x) + 3$$

$$x = 228 - 6x + 3$$

$$7x = 231$$

$$x = 33$$

larger = 33

smaller = 5

10. Jerry drives his truck 270 km in the same time George drives his car 250 km. If Jerry drives 4 km/h faster than George, what is each of their driving speeds?

	S	T	D
J	$x+4$	$\frac{270}{x+4}$	270

G	x	$\frac{250}{x}$	250
---	-----	-----------------	-----

$$\frac{270}{x+4} = \frac{250}{x}$$

$$270x = 250(x+4)$$

$$270x - 250x = 1000$$

$$20x = 1000$$

$$x = \frac{1000}{20}$$

$$x = 50 \text{ km/h}$$

11. The current of a river is 3 km/h. It takes a boat a total of 3 hours to travel a total distance of 24 km upstream and back. What is the speed of the boat in still water?

$$\therefore \text{Jerry} \rightarrow 54 \text{ km/h}$$

$$\uparrow \text{George} \rightarrow 50 \text{ km/h}$$